

Uncertainty analysis as a strategy to quality electrolytic sensors for measuring angular deviations

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Resumo. Este trabalho de pesquisa avalia a confiabilidade metrológica de sensores eletrolíticos utilizados para determinar desvios angulares de elementos de estruturas de engenharia civil. A metodologia considera a calibração simultânea de um conjunto de eletrômetros, permitindo mostrar que a expressão da incerteza associada à calibração constitui-se em fundamento essencial para assegurar a confiabilidade metrológica prestando-se, portanto, como método de descarte de eletrômetros fora da tolerância admissível para uma determinada aplicação. O tratamento estatístico da incerteza mostrou-se suficientemente robusto para validar o eletrômetro como instrumento fidedigno para medição de ângulo, com nível de confiança adequado à sua aplicação em engenharia civil.

Abstract. This work evaluates the metrological reliability of electrolytic sensors used to determine angular deviations of elements of civil engineering structures. The methodology considers the simultaneous calibration of a set of electrolevels, allowing to show that the expression of the uncertainty associated to the calibration is an essential fundament for assuring metrological reliability, thus providing a method of disposing of electrolevels outside the permissible tolerance for a given application. The statistical treatment of uncertainty was sufficiently robust to validate the electronic as a reliable instrument for angle measurement, with a level of confidence appropriate to its application in civil engineering.

1. Introduction

Among other applications, notably in Geotechnical Engineering, electrolevels are electrolytic sensors strategically used to monitor the behaviour and stability of various types of dams during their complex construction and operation processes (e.g.: rockfill dam; structural concrete dam; earth dam; tailings dam, intended to retain solid waste and water resulting from mineral extraction processes). In particular, they are also useful to identify critical regions of the dam exposed to deflections and distortions, which may introduce localized stress concentration. Considering that the electrolytic sensors are immersed in the material of the dam in different stages of the process of its construction or even of its operation, it is necessary to assure its metrological reliability before their installation. Based on the analysis of the mass of data that provides information associated with possible displacements of dam structures, it is possible to implement adjustments and corrections of the dam still during its construction process.

The continuous monitoring of the dam is also strategic during its operating (i.e. when subjected to full load), and can anticipate nonconformities that may result from unexpected overload of the dam structure as a whole, thus avoiding accidents, usually of unexpected proportions. Based on the uncertainty analysis associated with the measurement of angular displacements, this article proposes a strategy to assure the metrological reliability of electrolytic sensors, considering that its quality control must necessarily take into account not only the metrological rigor of the measurement process but also the stability of the electrolytic liquid and the integrity of the metal contacts during the electronics manufacturing process.

The careful metrological evaluation of measuring instruments (which includes the expression of associated uncertainties) is a vital tool for the development of any scientific experiment. To reduce the uncertainties associated with any measurement is an absolutely fundamental strategy to assign reliability to routine measurements (TAYLOR, 2012), thus reflecting the true physical meaning associated with the experiment.

2. Electrolytic sensors for measurement of angular deviations

Electrolytic sensors for measuring angular displacement are known as electrolevels. Widely used for level measurement in various engineering applications, electrolevels were originally designed for applications in the aeronautics industry, also finding application in civil and naval engineering and in the automotive industry. (Rocha Filho & Price, 2000).

2.1. Principle of operation of the electrolevel

Physically, electrolevel consists of an ampule of glass, plastic (Figure 1a) or ceramic material (Figure 1b), partially filled with an electrolytic liquid, interconnected to metallic connectors. Among the available options, it may have two or four coplanar electrodes, which penetrate the ampoule and are partially immersed in the liquid, forming what is called a Wheatstone Half Bridge (Figure 1c).

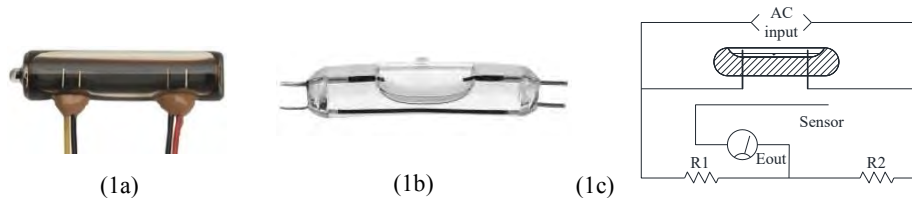


Figure 1 – Examples of different types of electrolevel and its powering electrical circuit

The electrolevel provides an output voltage that is proportional to its inclination angle, measured relative to a previously defined reference axis. The electrical impedance between the electrodes immersed in the electrolytic fluid varies as a function of the inclination imposed on the ampoule, reflecting the correlation that exists between the resistance variation of the electrode and the rotation of the set as a whole. The output signal of the electrolevel varies according to a range defined by the manufacturer based on its physicochemical properties. As a precaution, the electrolevel should not be excited by a DC voltage to prevent against electrolysis processes that adversely may affect the physical characteristics of the electrodes installed inside the ampule, immersed in the electrolytic fluid.

2.2. Fundamentals of the calibration procedure

The purpose of calibration is to adjust the response of an instrument so that it displays a true value relative to a traceable reference. Figure 2 illustrates a standard calibration procedure that aims to provide metrological reliability to the result of angular deviation measurement by means of an electrolytic sensor, fastened to a rigid rod, free to rotate relative to a fixed point. In the context of this calibration procedure, the angular deviation perceived by the electrolevel can be determined by the arc tangent of the angle φ (Equation 1), which is the angle generated by the rod when it rotates (around its axis of rotation) from the position of the reference axis to an arbitrary position, forming a triangle of catheter c and hypotenuse h , as defined in Figure 2. The metrological accuracy of the electrode calibration will therefore depend on (i) the metrological accuracy associated with the measurement of the vertical displacement (y) of the right end of the rigid bar and its length L and (ii) of the correctness of the measurement of the electrical potential difference (expressed by the calibration certificate of the voltmeter used), generated by the electrolevel output, before and after its rotation.

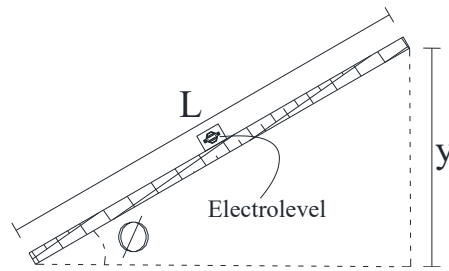


Figure 2 – Calibration procedure of a typical electrolevel

$$\phi = \arctan\left(\frac{y}{L}\right) \quad \text{Equation (1)}$$

Based on this fundamental calibration procedure, a Calibration Factor (CF) is defined by Equation (2), allowing the conversion of the output signal of the electrolevel to a measure of angular deviation. Figure 3a shows the results of the manufacturer's calibration (The Fredericks Company) of a single-axis electrolevel, Model 0715-4101-99 (Resolution: $0^{\circ} 0' 12''$), designed to operate within $\pm 9^{\circ}$.

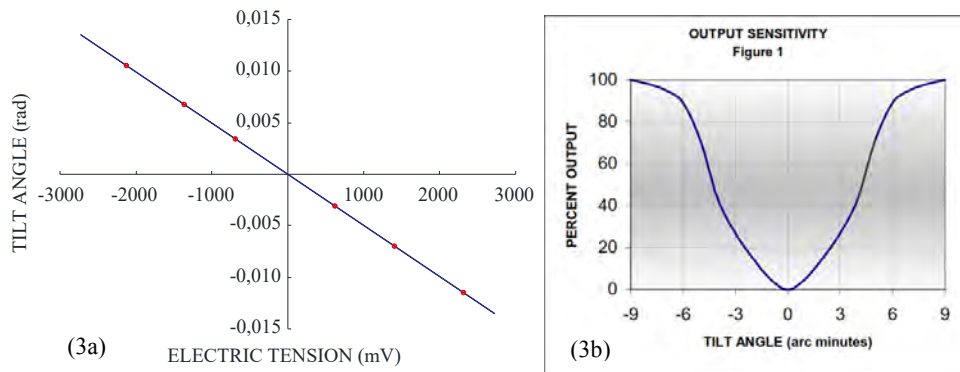


Figure 3 – Calibration of the reference electrolevel (3a: calibration curve; 3b: sensitivity response)

$$\theta \text{ (rad)} = (CF)_{reference\ standard} \cdot \Delta V \text{ (mV)} \quad \text{Equation (2)}$$

Where $(CF)_{padr\tilde{a}o} = 4,9 \cdot 10^{-6} \text{ rad/mV}$, derived from the calibration curve above, therefore allowing to convert the signal measured in millivolts in terms of angular deviation θ .

Figure 3b shows the performance of the electrolevel in its operating range of $\pm 9^{\circ}$, confirming, however, linearity of the signal in a smaller range, $\pm 3^{\circ}$.

3. Simultaneous calibration of a set of electrolevels by comparison to a standard

This section reports the results of Ramos (2009), regarding the simultaneous calibration of a set of nine electrolevels, performed by the method of comparison to the reference electrolevel, taken as a standard and whose calibration was described in the previous section.

3.1. Experimental setup

The images of Figure 4 illustrate the experimental arrangement assembled at the PUC-Rio Geotechnical Laboratory, used to simultaneously calibrate, by comparison to a standard of the same kind, the set of nine electrolevels. The Figure shows a schematic of the rigid metal bar, on which the nine electrolevels (E01 ... E09) were mounted, side by side, together with the standard (Reference electrolevel). In superimposed image, the same figure also shows a photograph of the assemblage, showing the cabling of each sensor, allowing monitoring of the output electrical signal that emerges from the *Wheatstone Half Bridge* to be measured by a calibrated voltmeter. In this experimental setup, each electrolevel was

carefully mounted inside an aluminium capsule, filled with epoxy resin, thus ensuring mechanical protection and absolute sealing of the sensitive elements.

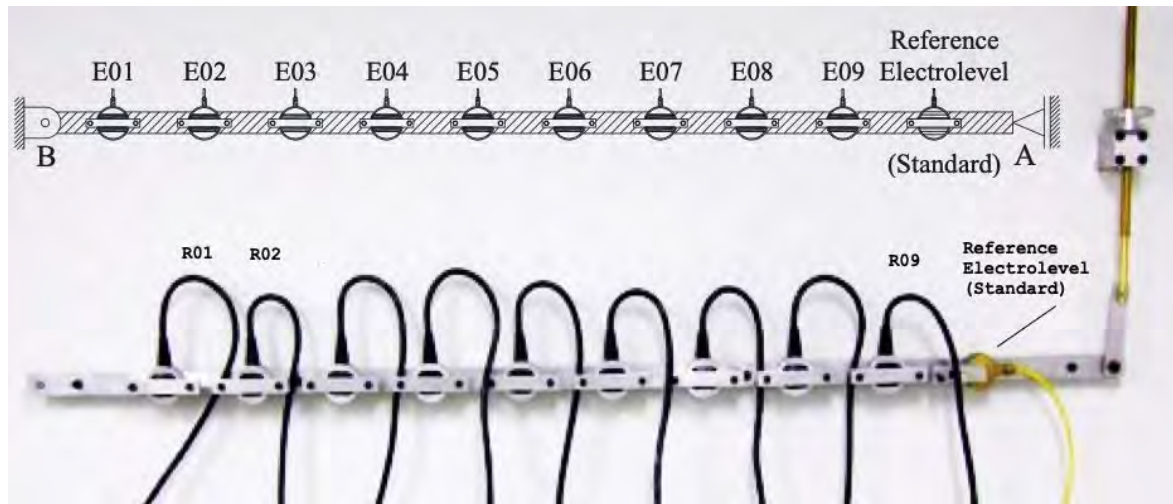


Figure 4 – Experimental set-up of the set of nine electrolevels mounted next to the reference standard

As can be seen in the photo above, at the opposite end of the fixed point of rotation, the metal rod has an endless thread device, which allows the bar angle to be varied, by means of a vertical displacement y , shown in Figure 4. The pitch of the thread is of 2.11 mm, and the length L of the rigid bar 1320 mm. Thus, an angular displacement of 0.0045 rad ($0^{\circ} 15' 0''$) is produced for every three turns of the thread.

3.2. Comparison calibration results

Table 1 summarizes the results of the simultaneous calibration of the nine electrolevels (Ramos, 2009), whose outputs are compared with the response of the reference electrolevel, calibrated by the manufacturer, in absolute compliance with the metrological rigor.

Table 1 – Results of simultaneous calibration of nine electrolevel (by comparison to the standard)

Experiment #	Reference electrolevel		Electrolevel output (measured values expressed in mV)								
	Tilt angle α (rad)	Electric tension (mV)	E01	E02	E03	E04	E05	E06	E07	E08	E09
1	-0.0199577	-4073	2814	2565	3137	2622	2642	2800	2969	2898	2816
2	-0.0149597	-3053	2106	1921	2345	1968	1966	2093	2215	2163	2108
3	-0.0100107	-2043	1398	1271	1563	1331	1298	1385	1459	1427	1393
4	-0.0050715	-1035	709	638	789	689	636	688	724	702	694
5	0	0	0	0	0	0	0	0	0	0	0
6	0.0046942	958	-706	-636	-779	-642	-661	-478	-746	-723	-706
7	0.0096824	1976	-1417	-1297	-1576	-1308	-1350	-1417	-1509	-1472	-1435
8	0.0147196	3004	-2134	-1952	-2366	-1951	-2029	-2134	-2263	-2218	-2162
9	0.0193893	3957	-2839	-2602	-3143	-2588	-2703	-2832	-3008	-2947	-2878

The slope angle θ is calculated by Equation (2), from the measured electrical voltage of the output signal of the reference electrolevel. In the same Table, the output for each electrolevel (expressed in mV) denotes the difference of two output signals associated with the angular positions of the rigid bar: when it is aligned with the reference axis (horizontal position, represented by experiment # 5 in Table 1) and when subjected to a rotation θ .

This is the reason why the output signal associated with the bar in position $\theta = 0$ indicates zero voltage values (values offset in relation to the output voltage, measured when the rigid bar is aligned with the reference axis; i.e. horizontal).

4. Assigning metrological credibility to the calibration process

This work aims to attribute credibility to measurement results not directly referenced to a direct calibration process, which is the case of the data reported by Ramos (2005), taken as the basis of a case study. Two fundamental premises support the development of this research: (i) although not directly referenced to a calibration standard, the data set available is consistent, therefore feasible to be statistically treated and (ii) a simultaneous calibration of multiple measuring devices (e.g.: electrolevels) might be considered of strategic interest, since the systematization of the calibration can result in saving time and research effort for civil engineering applications, without compromising the quality of the measurement results. In this context, the paper proposes a suitable formulation of a statistical tool to guide the quality control of electrolevels according to a strict criterion of permissible tolerances for geotechnical applications.

4.1. Fitting polynomials

The determination and application of the interpolation polynomial analysis that presents the least adjustment uncertainty, applied to each of the calibrations performed by Ramos (2009), allowed to correct the experimental results, not only eliminating the systematic error inherent to his measurements, but also allowing for interpolating values within the range of the applicable calibration. In accordance with good calibration practices (ISO GUM, 2008), polynomial analysis by the ordinary least squares method was repeated for each electrolevel and for three different degrees of the interpolating polynomial, therefore yielding 27 interpolating polynomials of the kind expressed in Equation (3). Although not documented in this paper due to space limitation, each one of these 27 polynomials are properly characterized and graphically represented in the master's dissertation of the first author (González Leño, 2019).

$$y(x_i) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 + \dots + a_n \cdot x^N \quad \text{Equation (3)}$$

4.2. Calculation of the measurement uncertainty associated with the polynomial fitting process

Following the proposed analysis, the data from the simultaneous calibration of the nine electrolevels allowed to calculate (i) the systematic error associated with each interpolator polynomial and (ii) the expanded uncertainty associated with the calibration of each of the electrolevels submitted to the simultaneous calibration process.

After applying each of the 27 polynomials (grades 1, 2 and 3) to the calibration data, which individually represents the raw data corresponding to each of the nine electrolevels calibrations, corrected values of the tilt angle are finally obtained, more precisely, the angular deviation measured relatively to a reference (horizontal) axis. Appendix A presents the overall results; i.e.: the calculated values of the angles adjusted by the correspondent interpolator polynomials, for each individual calibration of the nine electrolevels investigated.

Having obtained the adjusted values of the angles through the correspondent interpolator polynomial, which process data obtained from measurements of the output signal of each electrolevel, it is possible to calculate the uncertainty associated with the curve fitting (adjustment uncertainty). The final results of these calculations through the so called uncertainty propagation methodology (Equation 4) is summarized in Table 4.

The analysis of these results allowed to identify that, among the polynomials tested, and for the nine electrolevels simultaneously evaluated within the calibration experiments, the polynomial of degree 2 is the one that better models the physical nature of the calibration since it is polynomial that yields the lowest value of the adjustment uncertainty.

Table 4 – Identification of the polynomial that ensures the least adjustment uncertainty in the calibration

$$u_s = \sqrt{\left(\frac{1}{n-c}\right) \cdot \sum_{i=1}^n [y(x_i) - y_i]^2} \quad \text{Equation (4)}$$

Associated uncertainty of adjusted values									
Polynomial	E01 (rad)	E02 (rad)	E03 (rad)	E04 (rad)	E05 (rad)	E06 (rad)	E07 (rad)	E08 (rad)	E09 (rad)
Grade 1	0.00012762	0.00014619	0.00010399	0.00011642	0.00018021	0.00058623	0.00014067	0.00015457	0.00016732
Grade 2	0.00008417	0.00008927	0.00007379	0.00010994	0.00013392	0.00054269	0.00011486	0.00011545	0.00009934
Grade 3	0.00009366	0.00070222	0.00079388	0.00099707	0.00117032	0.00086259	0.00088884	0.00046375	0.00011019

From the number of degrees of freedom associated with the calibration performed (determined by the difference between the number of experimental points and the number of coefficients of the best polynomial fitting, i.e. 3, for the case of the polynomial of degree 2) and for a t-Student distribution of the calibration data, the Coverage Factor is stated to be $k = 1.96$, for a confidence level of 95% (ISO GUM 2008).

4.3. Calculated uncertainty associated with the calibration of each of the nine electrolevels

In accordance with the ISO GUM (2008), the uncertainty component is calculated by taking into account the three most important sources of uncertainties: (i) instrument resolution ($u_{inst} = 0.00005818$ rad); (ii) reference electrolevel ($u_{ref} = 0.00000490$ rad); (iii) polynomial fit (u_s). Table 5 documents the values of the uncertainty components obtained for each electrolevel associated with the calibration series. Thus, considering a Gaussian probability distribution for the experimental data, the expanded uncertainty (U_E) can be estimated for the stated Coverage Factor ($k = 1.96$), as detailed in Table 5 below.

Table 5 – Assessing the uncertainty analysis

$$u_c^2 = u_{inst}^2 + u_p^2 + u_{ref}^2 + u_s^2 \quad \text{Equation (5)}$$

$$U_E = u_c \cdot k \quad \text{Equation (6)}$$

Electrolevel	Uncertainty associated with the resolution	Uncertainty associated with the reference electrolevel	Uncertainty associated with the adjustment	Combined uncertainty	Expanded uncertainty
	u_{inst} (rad)	u_p (rad)	u_s (rad)	u_c (rad)	U_E (rad)
E01	0.0000168	0.0000014	0.0000842	0.0000858	0.0001683
E02	0.0000168	0.0000014	0.0000893	0.0000908	0.0001781
E03	0.0000168	0.0000014	0.0000738	0.0000757	0.0001484
E04	0.0000168	0.0000014	0.0001099	0.0001112	0.0002180
E05	0.0000168	0.0000014	0.0001339	0.0001350	0.0002646
E06	0.0000168	0.0000014	0.0005427	0.0005429	0.0010642
E07	0.0000168	0.0000014	0.0001149	0.0001161	0.0002275
E08	0.0000168	0.0000014	0.0001154	0.0001167	0.0002287
E09	0.0000168	0.0000014	0.0000993	0.0001008	0.0001975

As can be observed, except for the electrolevel E06, all the experimental results associated with the calibration of the other eight electrolevels present excellent agreement. The results of this round of experiments were grouped and shown in Figure 5, whose horizontal axis denotes the adjusted angle calculated on the basis of the best interpolator polynomial and the vertical axis, denote the sum of the systematic error and the expanded uncertainty, respectively for each data of calibration. For the analyzed data set, it is observed that the systematic error added to the expanded uncertainty does not exceed the

value 0.0005 rad, understood as convenient threshold limiting value that could be used to define the tolerance of the technological application of geotechnical interest.

Figure 5 illustrates the graphical representation of the systematic error added to the expanded uncertainty (defined by some authors as the “total error”) associated with the measurements.

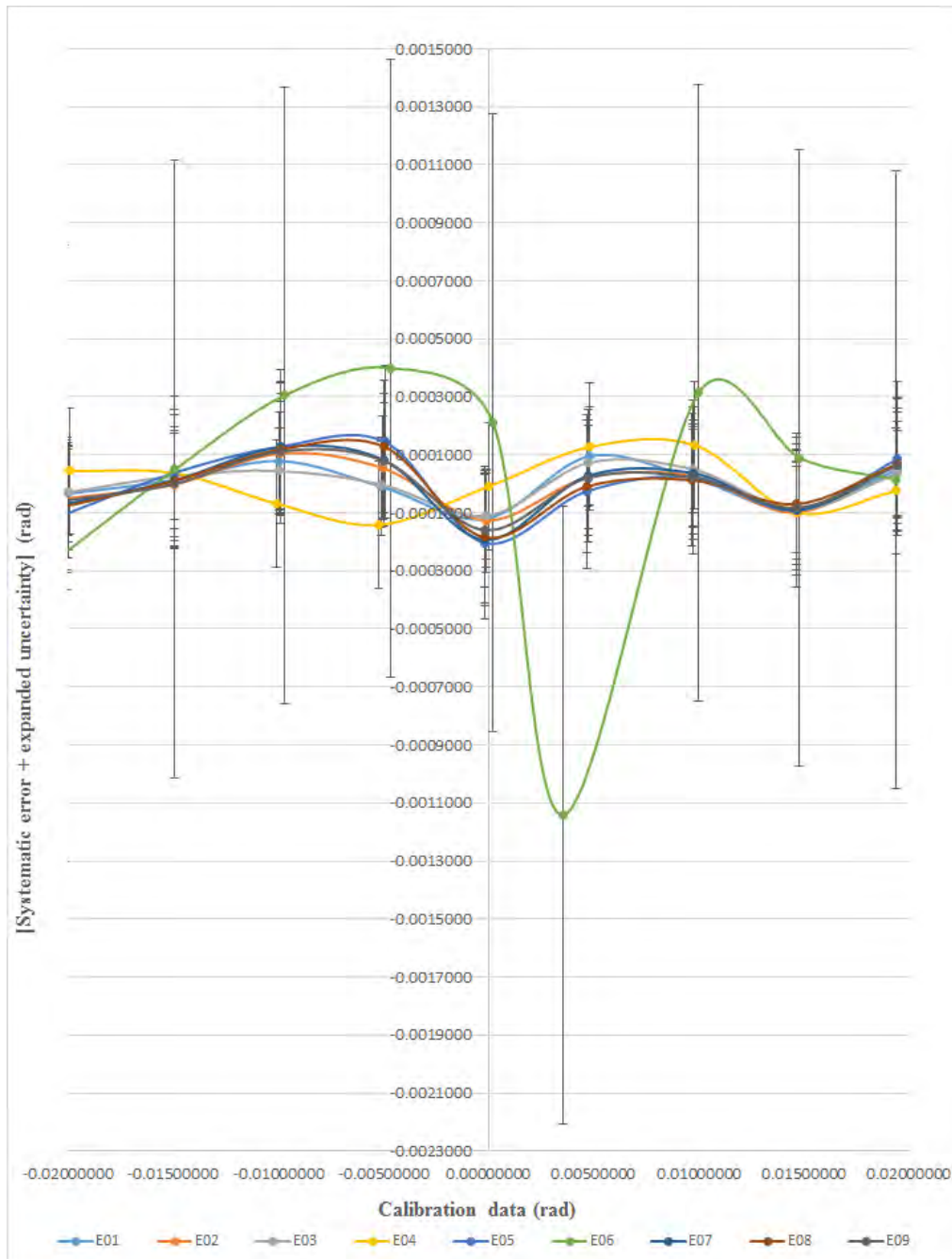


Figure 5 – Systematic error added to the expanded uncertainty associated with the calibration data

As shown, the electrolevel E06 exhibits an abnormal behavior (whose calibration exhibits a much larger measurement uncertainty) when compared to the other eight equally calibrated electrolevels. This a typical performance acts as a disposal criterion, indicating that the E06 sensor is not suitable for use since it does not conform to the level of the allowed tolerance.

Among the possible explanations for the behavioral deviation of the E06 electrolevel response, the following causes could be charged: carelessness in the operation of the instrument; possibility of outliers being considered during the measurement process; oxidation of the electrolytic fluid, deterioration of electrical contacts between the electrolytic fluid and the metallic electrodes, micro-rupture of the capsule, all possible reasons which may compromise the integrity of the sensitive element as a whole. Until each of these causes is individually investigated, it is recommended not to make any value judgment and simply discard the electrolevel that exhibits this type of behavior.

5. Conclusions

The proposed measurement uncertainty assessment methodology proved to be effective in assuring the metrological reliability of the simultaneous calibration process of a set of nine electrolevels, ensuring traceability to the International System of Units by comparison to a calibrated standard of angular deviation measurements. The method tested to assess the robustness of a typical calibration performed by professionals in the field not only attributes credibility to the reported results because it incorporates to the measurement results their associated uncertainties, but, also, proved to be a strategic robust disposal criterion to refuse marketed electrolevels that fail to meet acceptable standards of tolerance, compatible with specific applications of interest in Geotechnical Engineering.

References

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Appendix A: Tilt angles corrected through the calibration polynomials

Polynomial	E01	E02	E03	E04	E05	E06	E07	E08	E09
	Corrected values of the tilt angles calculated from the 27 calibration polynomials tested (rad)								
Grade 1	-0.0198494	-0.0198378	-0.0198770	-0.0198341	-0.0198763	-0.0197674	-0.0198951	-0.0198748	-0.0198169
	-0.0149111	-0.0149189	-0.0149064	-0.0149036	-0.0148759	-0.0147979	-0.0149150	-0.0149091	-0.0149097
	-0.0099728	-0.0099542	-0.0099986	-0.0101013	-0.0099347	-0.0098214	-0.0099216	-0.0099367	-0.0099541
	-0.0051670	-0.0051193	-0.0051410	-0.0052613	-0.0050379	-0.0049222	-0.0050669	-0.0050386	-0.0051093
	-0.0002217	-0.0002463	-0.0001892	-0.0000669	-0.0003334	-0.0000862	-0.0002849	-0.0002959	-0.0002992
	0.0047027	0.0046115	0.0046998	0.0047732	0.0045560	0.0032736	0.0046424	0.0045887	0.0045941
	0.0096619	0.0096602	0.0097018	0.0097941	0.0096526	0.0098739	0.0096820	0.0096489	0.0096468
	0.0146630	0.0146631	0.0146598	0.0146417	0.0146751	0.0149137	0.0146622	0.0146889	0.0146856
	0.0195803	0.0196278	0.0195363	0.0194441	0.0196607	0.0198199	0.0195829	0.0196140	0.0196482
Grade 2	-0.0199914	-0.0200075	-0.0199865	-0.0199134	-0.0200602	-0.0201975	-0.0200258	-0.0200333	-0.0200142
	-0.0149463	-0.0149619	-0.0149328	-0.0149229	-0.0149209	-0.0149100	-0.0149476	-0.0149492	-0.0149603
	-0.0099319	-0.0099059	-0.0099664	-0.0100787	-0.0098820	-0.0097061	-0.0098843	-0.0098922	-0.0098988
	-0.0050814	-0.0050175	-0.0050740	-0.0052130	-0.0049271	-0.0046719	-0.0049889	-0.0049441	-0.0049915
	-0.0001203	-0.0001257	-0.0001104	-0.0000095	-0.0002032	0.0002107	-0.0001929	-0.0001844	-0.0001594
	0.0047893	0.0047155	0.0047669	0.0048217	0.0046686	0.0035524	0.0047216	0.0046852	0.0047152
	0.0097030	0.0097097	0.0097328	0.0098161	0.0097058	0.0099961	0.0097195	0.0096953	0.0097051
	0.0146269	0.0146211	0.0146310	0.0146210	0.0146290	0.0148085	0.0146300	0.0146502	0.0146375
	0.0194379	0.0194581	0.0194253	0.0193646	0.0194757	0.0194037	0.0194542	0.0194584	0.0194523
Grade 3	-0.0199464	-0.0207144	-0.0190571	-0.0188620	-0.0186044	-0.0192275	-0.0209499	-0.0204946	-0.0200282
	-0.0149377	-0.0155447	-0.0142542	-0.0140159	-0.0139646	-0.0144785	-0.0157162	-0.0153802	-0.0150494
	-0.0099337	-0.0103121	-0.0095192	-0.0094096	-0.0093097	-0.0095614	-0.0104307	-0.0102210	-0.0100043
	-0.0050767	-0.0052177	-0.0048455	-0.0048490	-0.0046630	-0.0046456	-0.0052754	-0.0051253	-0.0050743
	-0.0001000	-0.0001000	-0.0001000	-0.0000100	-0.0002000	0.0002000	-0.0002000	-0.0002000	-0.0002000
	0.0048256	0.0049692	0.0045609	0.0044810	0.0044037	0.0035211	0.0050075	0.0048415	0.0047189
	0.0097474	0.0101869	0.0092985	0.0091565	0.0091279	0.0098245	0.0102900	0.0100071	0.0097360
	0.0146625	0.0152892	0.0139576	0.0137194	0.0136713	0.0143394	0.0154459	0.0150693	0.0146561
	0.0194397	0.0202690	0.0184983	0.0183187	0.0180338	0.0183936	0.0204573	0.0199125	0.0194001