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The use of Electro-levels for monitoring a vertical pile subjected to horizontal load- FMGM2018

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SUMMARY: In the study of deep foundations subjected to lateral forces, it is very important to know how foundation loads are transmitted to the supporting soil and to observe the soil strains and displacements around the loaded pile. This paper describes the use of Electro-levels to analyze the interaction between the pile and the soil, in terms of displacements, rotations, bending moment, shear and soil reaction (stress) during the loading test of the instrumented pile. The behavior of a vertical pile subjected to horizontal loads was analyzed using Electro-levels placed at different locations in the pile shaft. Electro-levels can provide accurate values of the rotation of the pile cross section. The results obtained from the instrumentation, adjusted in the form of polynomials (sequential analysis), show the advantage of obtaining the parameters of interest in the project through the Winkler model with the ease of deriving the rotation function obtained. The least square method, with restrictions established with the Lagrangian multiplier method, was used to adjust the data. The limitations of the method based on the Winkler model are highlighted; the mathematical formulation requires parameters from the derivation process with a consequent expansion of errors to obtain the parameters of interest. From the experimental point of view, special attention must be given to the non-linearity of the increase of the loadings applied to the pile. The objective was to emphasize the interpretation of experimental results obtained in a monitored laterally loaded pile using Electro-levels. By interpreting the results, the behavior of a pile subjected to horizontal loading was analyzed and the parameters of interest obtained are convincing and therefore satisfactory.

KEYWORDS: Instrumentation, Electro-levels, Pile, Horizontal loading

1 INTRODUCTION

In any civil engineering work involving structural elements in contact with soil, distinct aspects of both geotechnical and structural engineering have to be considered. However, the soil-structure interaction will determine the success of the project as a whole.

In the study of the load transfer mechanism, the relative stiffness (i.e., stiffness of the structural element compared to the stiffness of the soil) in some cases, is the main factor to take in account when considering the stress-strain characteristics in soil-structure interaction.

The analysis of a vertical pile embedded in a soil will dependent on the consolidation of the soil, which is fundamental for a soil-structure interaction. In the case of vertically loaded piles, the pile will experience displacements in the same direction of the applied load and the induced strains in the soil adjacent to the pile will present compatible values along the shaft of the pile. The behavior of this pile will be determined predominantly by the geotechnical aspect and not according to the performance of the pile material (except in the case of bending of piles supported in firm layers).

In contrast, for the case of horizontal loading, the largest displacements will occur in the upper third of the pile, and within that region, the induced strains in the soil adjacent to the pile will vary by an order of magnitude. The behavior of a laterally loaded pile will be determined by the interaction between the structural foundation and the adjacent soil. In this case, the behavior of the pile material should also be considered in the analysis, since this will be subject to large bending moments.

Pile load tests of lateral load capacity are commonly used to obtain the horizontal displacements of the pile or, more specifically, to measure the variation of the displacement as the depth increases. The response of the soil to any lateral load is non-linear. This implies that the value of the modulus of elasticity is not constant with the depth or with the level of load acting on the soil, consequently, both aspects have to be adequately considered to analyze the lateral response of piles.

2 HORIZONTAL SOIL REACTION THEORY

Figure 1 shows a vertical pile subjected to a horizontal force on the top and the soil reaction, for that situation the pile will act as a thin strip whose behavior is governed by the beam equation, which was originally proposed by Hetenyi (1946). For a beam on an elastic foundation (also known as soil reaction method), the governing differential equation is given as [Poulos and Davis, 1980; Saran, 2009]:

$$EI\frac{d^4y}{dz^4}+k_h y=0 \qquad (1)$$

where:

E = Young's modulus of pile material
I = Moment of inertia of the pile
k_h = Soil reaction modulus
y = displacement

Assuming that the pile act as a thin strip whose behavior is governed by the beam equation in an elastic medium, the rotation of the cross section of the pile is obtained through the first derivative of the displacement equation as shown in Figure 2. The second derivative of the displacement (y) equation multiplied by the modulus of elasticity (E) and moment of inertia (I) of the pile will provide the bending moment along the shaft of the pile. The mathematical process of derivation can increase the possible errors involved in the process of experimentally obtaining the variation of the pile displacements with depth.

Electro-levels provide direct rotation data with high precision; however, the other stresses acting along the shaft of the pile are obtained by derivation or integration of the adjusted function and are susceptible to error. The direct measurement of the rotation of the cross section of the pile reduces one operation of derivation, so bending moments are obtained with great accuracy.

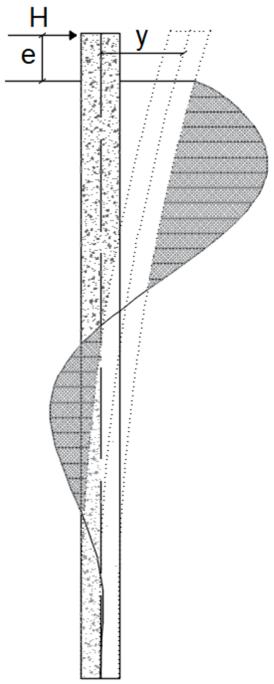

Figure 1. A vertical pile subjected to a horizontal force and the soil reaction.

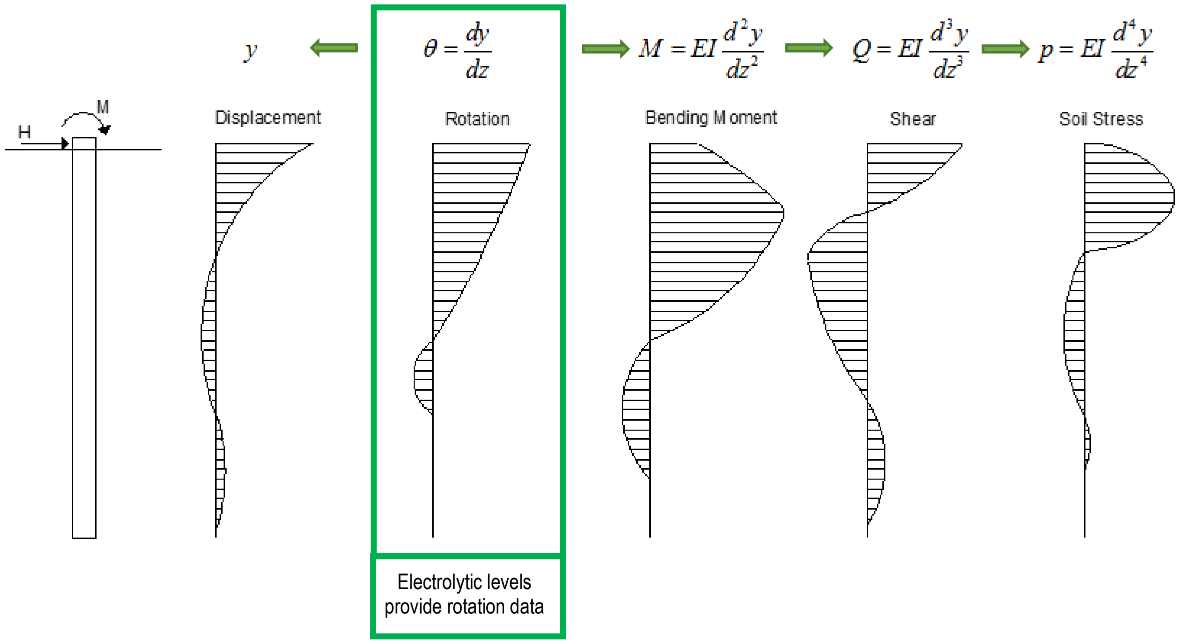

Figure 2. Complete solution for a vertical pile subjected to a horizontal force.

In this work, a restriction on the extremes of the adjusted function was applied in order to minimize the errors in the derivatives. In many practical cases, the design of piles subjected to horizontal loads requires that the horizontal load-displacement limit be satisfied, a requirement that may result in the specification of permissible lateral loads much smaller than the maximum lateral load capacity of the piles. The solution of such problems usually involves the use of iterative techniques because the soil reaction is a nonlinear function of the structural displacement. Experimental investigations and advances in analytical techniques have resulted in a more general understanding of the problem, making it possible to include the properties of the pile and the non-idealized behavior of the soil in the analysis.

The soil reaction is how soil resists the action of the pile, is a complex subject depending on soil type, loading level, pile geometry, etc. The soil around a horizontally loaded pile is requested in compression on one side and in tensile on the other side; on the tensile side, the soil tends not to follow the pile.

2.1 Winkler model

The analysis by load transfer function for laterally loaded piles, also known as horizontal soil reaction theory, is based on the Winkler model (1867). It is assumed that the beam is supported by a soil model in which the elastic continuum foundation is replaced by a series of elastic independent springs as shown in Figure 3. Assuming that theory, deformations in the soil will occur only where loading exists, this analysis relates the load is transferred to the soil at any depth along the pile, with the displacement at that depth.

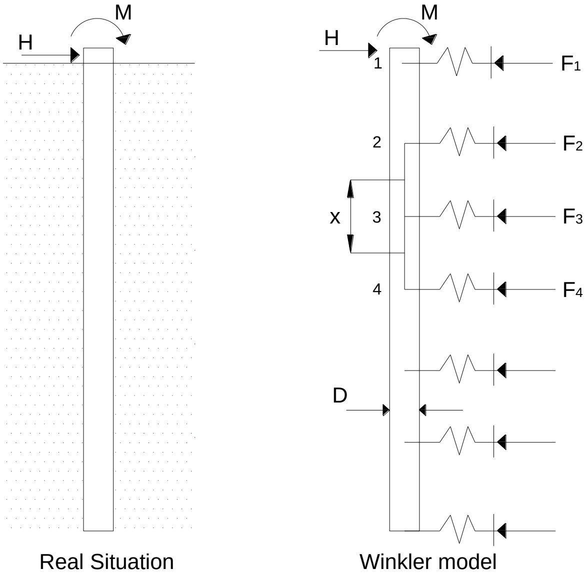

Figure 3. The Winkler model

Among the disadvantages of this model is the lack of consideration of soil continuity, and the fact that the stiffness of each spring depends on the dimensions of the foundation. Despite these obstacles, soil reaction theory is widely used in practice for foundations, since it the approximation of the soil rigidity with the depth and profiles of non-homogeneous soils.

When the transfer of a unit load and displacements at various depths is known, it is possible to construct a load transfer function for each segment of the pile, called as $p-y$ curves for laterally loaded piles. That $p-y$ curves allow considering different levels of mobilization at each depth, depending on the displacements of the pile, and even the rupture of the surface material. Once such functions are established the process of designing a pile in a similar soil is merely the inverse of this analysis.

To obtain solutions for the soil-structure system that satisfy conditions of equilibrium and compatibility, the analysis should include variations of soil properties with depth, stress relations, non-linear deformations and three-dimensional considerations. However, it is difficult to analyze the soil-pile interaction to satisfy all the necessary requirements; thus, only the behavior of a vertical pile subjected to lateral load on the top is presented in this paper.

2.2 Horizontal reaction coefficient of the soil (k_h)

Terzaghi in 1955, define the soil reaction coefficient, k_s, of a soil at a given depth (z) as the relationship between the unit stresss (σ_z) acting at that depth (z) and the displacement (y) suffered by the soil as shown in Figure 4 and Equation 2, expressed in units of force per length raised to the cube. The value of k depends on the elastic properties of the soil and the dimensions of the area being loaded and should not be considered as a constant value for every soil.

$$k = \frac{\sigma_z}{y} \quad [FL^{-3}] \qquad (2)$$

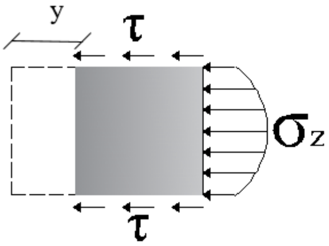

Figure 4. Plan view of a distribution of soil reaction at the shaft of the pile at a given depth z

Figure 4, represents the stress distribution on the face of a pile element that has experienced a constant, horizontal, displacement (y). Since the pile is considered to be rigid in the horizontal plane compared to the ground, the stress distribution (σ_z), is not constant along the face in contact with the ground. In this case, the soil reaction acts in the horizontal direction; the relation between the stress and displacement is called as horizontal reaction coefficient (units FL^{-3}), k_h given by:

$$k_h = \frac{\sigma_z}{y} \qquad (3)$$

Considering that the resultant of the stresses acting in an area corresponding to the front of the pile, that is, in a strip with an equal width to the diameter or width of the pile, called as D. The soil reaction is a normal stress (p), acting in the width D perpendicular to which the horizontal displacement occurs, thus we have:

$$p = k_h y \qquad (4)$$

Where:

p = normal stress (FL^{-2})
k_h = Horizontal reaction coefficient of the soil (FL^{-3})
y = Displacement

Assuming that stress is constant along the face of width “D” of the pile and considering as “K” the Horizontal reaction coefficient of the soil incorporating the transversal dimension of the pile, we have:

$K = \frac{\sigma_z D}{y}$ (5)

Where:

K = Horizontal reaction coefficient of the soil incorporating the transversal dimension of the pile “D” (FL^{-2})
D = width of the pile

Replacing equation 3 in equation 5, we have:

$K = k_h D \quad [FL^{-2}]$ (6)

The modulus of elasticity of the soil depends on the drainage conditions, type and level of loading, and the Horizontal reaction coefficient of the soil may be obtained from a) lateral load tests on piles in real scale, b) plate load tests, c) empirical correlations with other soil parameters, d) field tests, e) reduced model tests. In the international bibliography the horizontal reaction coefficient is also mentioned as modulus of subgrade of the soil.

For practical purposes, Brooms (1964) and Pyke & Beiake (1985) suggest approximating its value (k_h) as shown in Equation 7, however, the use of this correlation in practical cases is limited.

$k_h = \frac{E}{D}$ (7)

In most conventional tests, only the load and the horizontal movement at the top of the pile are measured. However, this information is sufficient only for a critical overview of the project; it is not possible to establish the horizontal deformation caused by the load along the entire pile shaft and then extrapolate these results to piles with different geometries.

In order to obtain more detailed information about the performance of the pile with depth, the use of inclinometers is often presented in the technical literature to measure the displacements along the shaft of the pile. However, in the analysis of these results greater errors occur from the process of derivation of the adjusted function, in order to establish the bending moments and the reaction of the soil. The bending moments can also be measured by means of extensometers or the soil reaction through total pressure cells. The analysis of the results of this type of instrumentation requires a smaller number of derivates and, consequently, lower error propagation. On the other hand, this type of instrumentation requires greater care in the handling, transportation, and installation process, with an increased danger of damaging the sensors during the installation of the pile itself.

Price & Wardle (1987a, 1987c, 1987d) presented a very attractive alternative that consists of the direct measurement of the rotation of the pile using Electro-levels. This instrumentation system can be installed in tubes, not necessarily attached to the pile, so it can be used in other soil-structure interaction activities, and can be reused in other tests, reducing the real cost of the instrumentation. In this work, using that methodology in order to obtain the modulus of subgrade of the soil in situ k_h, geotechnical instrumentation was employed to measure stresses in the soil and displacements in the pile along its entire length. However, this type of test requires special care in the instrumentation process, as well as in the interpretation of the results, therefore is rarely performed in practice. An inaccurate but frequently adopted process in practice is to measure the displacements at the top and to recalculate the value of k_h by assuming an appropriate distribution with depth.

Price & Wardle (1987b) instrumented a pile using vibrating cord extrensometers; Ramos (1988) presented a comparison of these results and the same instrumented pile using both Electro-levels and vibrating cord extrensometers, concluding that the results were approximately the same. In this work, the results provided by the Building Research Establishment (Price & Wardle 1987b) were used to apply the use of polynomial functions to interpret data of the instrumentation program provided.

3 ELECTRO-LEVELS

As shown in figure 5, electro-levels consist of short lengths of glass tubing into which an electrolyte is sealed. A typical tilt sensor has three electrodes: a common electrode and two outer electrodes, electrodes near each end and at center of the tube are partially immersed in the electrolyte and the resistance between the central electrode and the end ones varies as the tube is tilted in the vertical plane containing the tube. By connecting, the levels to an AC strain indicator or in an AC bridge circuit with a precision voltmeter very small change in inclination to the horizontal can be measured. Electro-levels provide an output voltage that is proportional to the tilt angle of the sensor with reference to gravity.

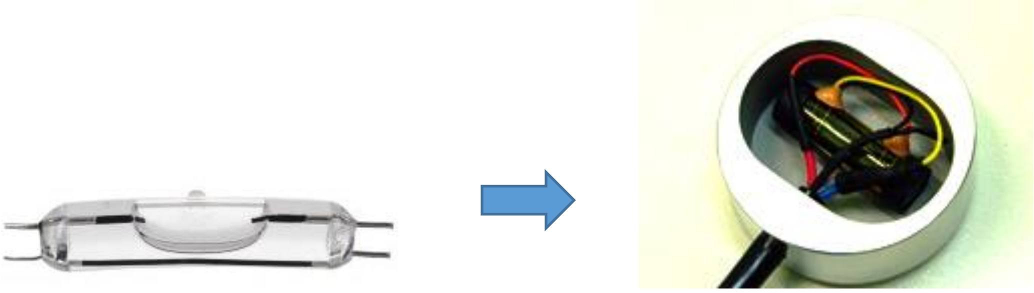

Figure 5. a) Electrolytic level (www.fredrickscompany.com) b) Capsule assembly of the Electrolytic level

4 DESCRIPTIONS OF THE FORMULATIONS FOR PILE AND SOIL ANALYSIS

The Lagrange multiplier method was used to minimize the oscillations that occur at the extremes of functions adjusted by the least squares' method. It is necessary to identify the possible derivatives of the rotation function obtained from the measurements of the rotations along the pile shaft, so that the other stresses can be determined.

Assuming $y(x)$ is a polynomial of n degrees of rotation, and $y'(x)$, $y''(x)$ e $y'''(x)$ as their derivatives representing, respectively, the bending moments, shear forces and soil reaction, we have:

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n \qquad (8)$$

$$\frac{M}{EI} = y'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \ldots + n a_n x^{n-1} \qquad (9)$$

$$\frac{Q}{EI} = y''(x) = 2a_2 + 6a_3 x + \ldots + n(n-1) a_n x^{n-2} \qquad (10)$$

$$\frac{p}{EI} = y'''(x) = 6a_3 + \ldots + n(n-1)(n-2) a_n x^{n-3} \qquad (11)$$

By accepting the following additional conditions, for the first and second derivatives:

$$\varphi = y'(x) - \frac{M_o}{EI} = 0 \qquad (12)$$

$$\varphi = y''(x) - \frac{Q_o}{EI} = 0 \qquad (13)$$

$$\varphi = y'(x) - \frac{M_k}{EI} = 0 \qquad (14)$$

$$\varphi = y''(x) - \frac{M_k}{EI} = 0 \qquad (15)$$

the deviations between the adjusted function and the points supplied shall be such that:

$$\theta = \sum_{i=1}^{n} (y_i(x) - y_i)^2 + \lambda_1 \left(y'(x_0) - \frac{M_o}{EI} \right) + \lambda_2 \left(y''(x_0) - \frac{Q_o}{EI} \right) + \lambda_3 \left(y'(x_k) - \frac{M_k}{EI} \right) + \lambda_4 \left(y''(x_k) - \frac{Q_k}{EI} \right) = 0 \quad (16)$$

Deriving each term from the above expression as a function of the variables, we obtain the system of equations:

$$\tilde{X} \cdot \tilde{a} = \tilde{b} \qquad (17)$$

where:

$$\tilde{X} = \begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 & \dots & \sum x_i^n & 0 & 0 & 0 & 0 \\ & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \dots & \sum x_i^{n+1} & 1 & 0 & 1 & 0 \\ & & \sum x_i^4 & \sum x_i^5 & \dots & \sum x_i^{n+2} & 2x_o & 2 & 2x_k & 2 \\ & & & \sum x_i^6 & \dots & \sum x_i^{n+3} & 3x_o^2 & 6x_o & 3x_k^2 & 6x_o \\ & & & & & \dots & \dots & \dots & \dots & \dots \\ & & & & & \sum x_i^{2n} & \dots & \dots & \dots & \dots \\ & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & 0 \end{bmatrix} \qquad (18)$$

$$\tilde{a} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \dots & a_n & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}^t \qquad (19)$$

$$\tilde{b} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i^2 \\ \sum y_i x_i^3 \\ \cdots \\ \sum y_i x_i^n \\ \dfrac{M_o}{EI} \\ \dfrac{Q_o}{EI} \\ \dfrac{M_k}{EI} \\ \dfrac{Q_k}{EI} \end{bmatrix} \qquad (20)$$

The following restriction was adopted: bending moment and shear are equal to the load applied at the ground level and equal to zero at a given pile depth. At ground level, the moment and shear are, respectively, equal to H and $H \cdot x$, where H is the horizontal load at the pile and x is the distance between the load and the ground level. The bending moment and the shear are practically null at the tip of the pile.

5 ANALYSIS OF EXPERIMENTAL RESULTS

Suitable polynomial functions for the interpretation of results were investigated, to obtain the parameters of interest of the soil.

5.1 Description of the pile instrumentation system

According to the provided data and complementary information found in the Ramos (1988) publication, the tests were conducted at Canons Park, London, where the site soil is the London clay, soil that has been extensively studied by different researchers. In the international literature, there is much data about the properties of London clay.

The tests were carried out in a vertically driven pile that was cast-in-place, with 0.17 m in diameter and had embedded length of 4.5 m as shown in Figure 6.

The rotations were measured using Electro-levels with great accuracy (1/206265 rad). Ten Electro-levels with a vertical spacing of about 200mm, were installed in a plastic tube placed inside the pile, with the lowest electro-level at the base of the tube and the others successively stacked on top. In addition, other tubes were installed at a radial distance of 100 and 300 mm from the pile with a series of five Electro-levels spaced approximately 400 mm apart vertically. The horizontal movement of the pile and tubes above ground level was measured by LVDTs. The results are satisfactory for all load levels.

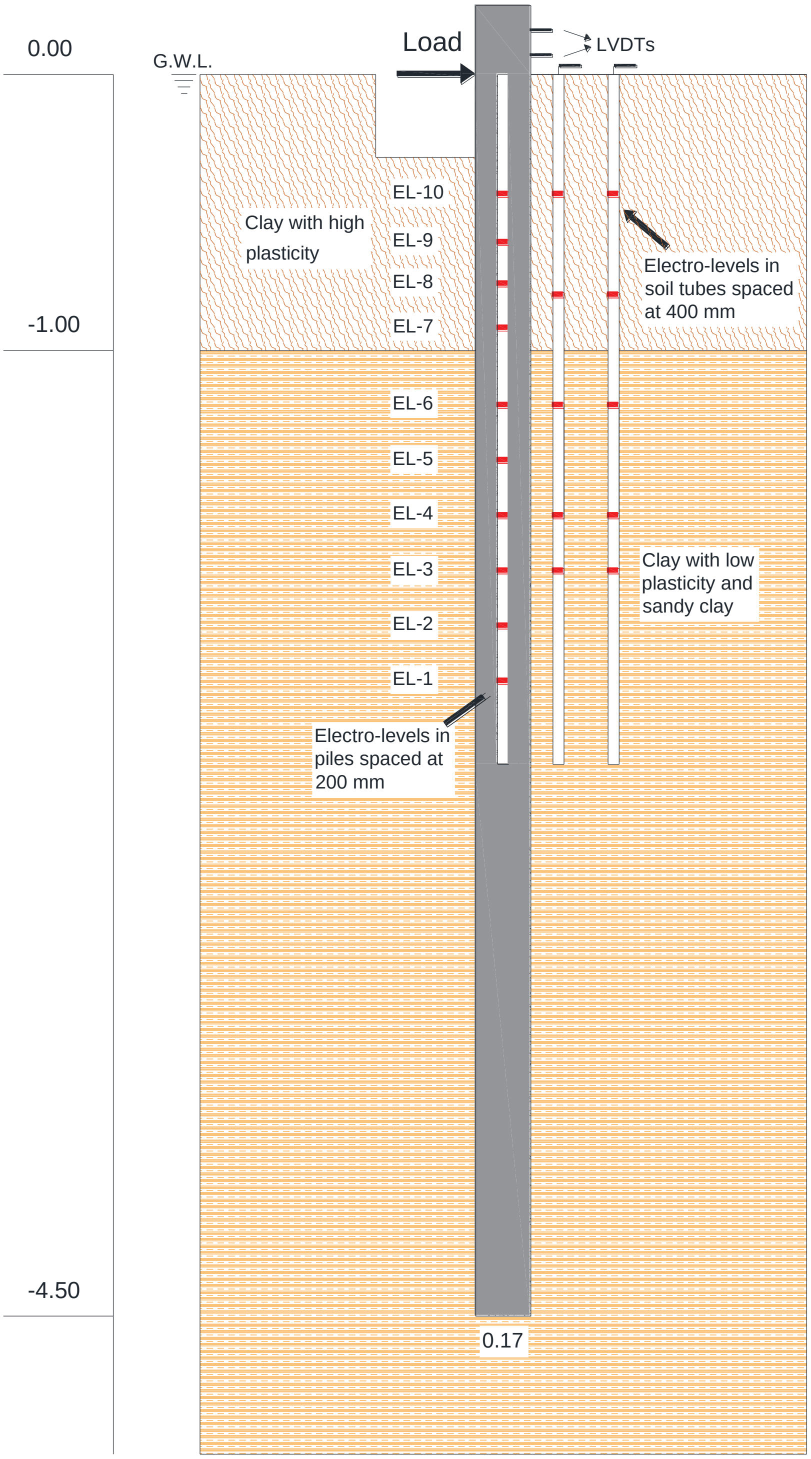

Figure 6. Details of the instrumented pile

5.2 Rotations

In order to check for any disturbances that may have occurred to the Electro-levels during the installation process, the rotations in each electro-level were measured before the application of lateral loads. The results obtained were taken as a reference for all loading levels. Subsequently, load tests were performed with loads of a) 0.0242 kN, b)1.4278 kN, c) 2.4684 kN, d) 3.6058 kN, e) 4.5254 kN and f) 5.2030 kN. In the present work, due to the large amount of data, only the analyses of results for the load of 5.2 kN are presented.

The results from the electro-levels were checked by plotting the rotations against the horizontal loads. Figure 7 shows the almost linear behavior of the horizontal load x rotation graphs for the 10 Electro-levels. An interesting observation is that from their position of installation in the pile, it was expected that the rotations measured by electro-level 9 would be larger than those measured by electro-level 8. This did not occur, leading us to conjecture that there may have been some change in the behavior of the soil at that depth, or there may have been some disturbance during the installation process of electro-level 9. It was also observed that the results of electro-levels 1 and 2 showed almost null rotations compared to the others, leading to the supposition that these results should exert little influence on the analyzes. Based on these results, equations were established indicating the variation of the pile rotations with depth, for the different levels of lateral loading.

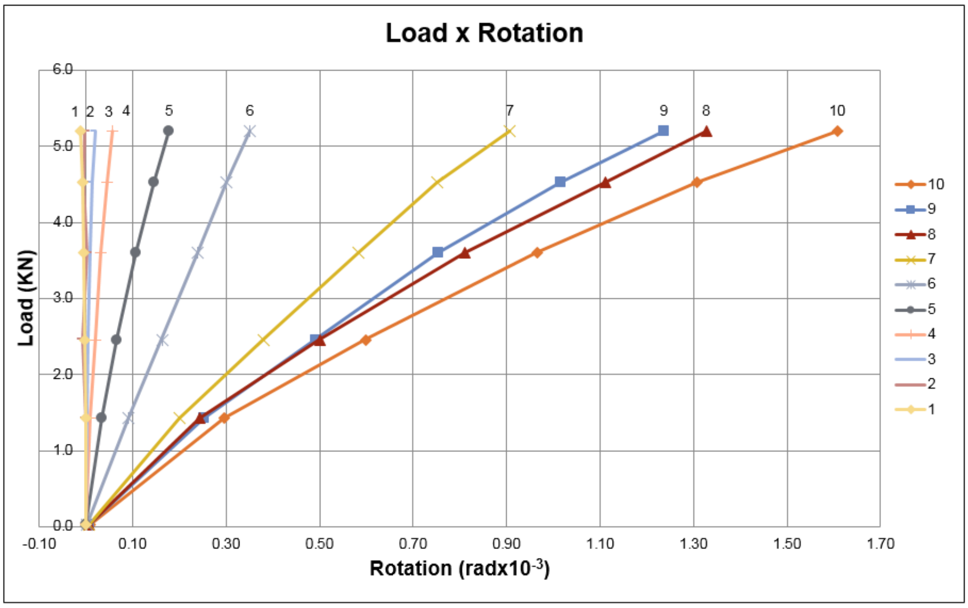

Figure 7. Behavior of the pile with the horizontal load applied and rotations measured

5.3 Horizontal displacement and specific deformation of the pile

As an immediate application of the results of the electro-levels, the horizontal displacements and the specific deformations can be established in a pile element using a model of angular distortion and displacement. The horizontal movement of the pile can be determined by adding the displacements calculated for each electro-level, starting with the electro-level at the base of the pile (Figure 8). Adopting this calculation, the variation of the rotation θ measured by each electro-level, is applied to half the distance x for each of the adjacent electro-levels. The displacement d over the distance x is then calculated as follows:

$$d = \tan(\theta) \cdot x \qquad (21)$$

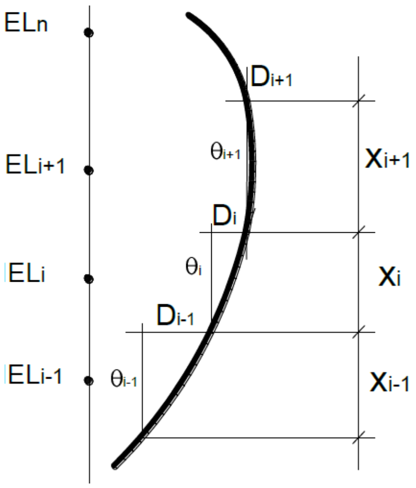

Figure 8. Calculations of horizontal displacements from the electro-levels' readings

The initial position of the lower electro-level should be adopted for half the distance between it and the adjacent electro-level. For the top electro-level, it was sufficient to measure the pile rotation at the ground surface, or at a position where the horizontal movement of the pile above ground level is known.

The horizontal displacements at ground level were measured with LVDTs installed in the pile and in the tubes. The values of the horizontal displacements calculated from the results of the Electro-levels for the pile and for the tubes are presented in Figures 9, 10 and 11.

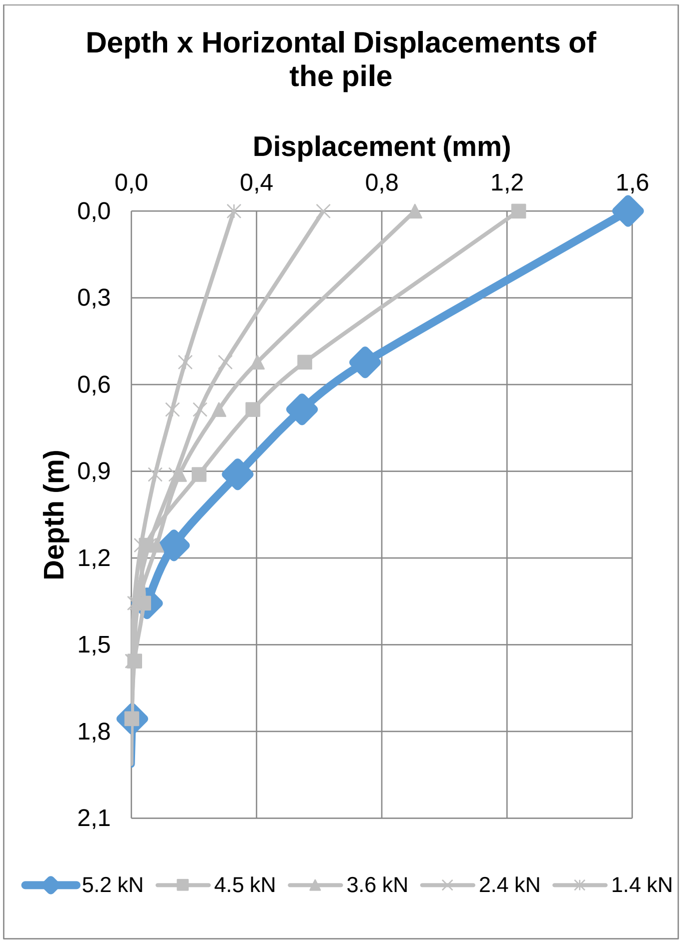

Figure 9. Horizontal displacements of the pile horizontally loaded with 5.2kN; results of the electro-levels

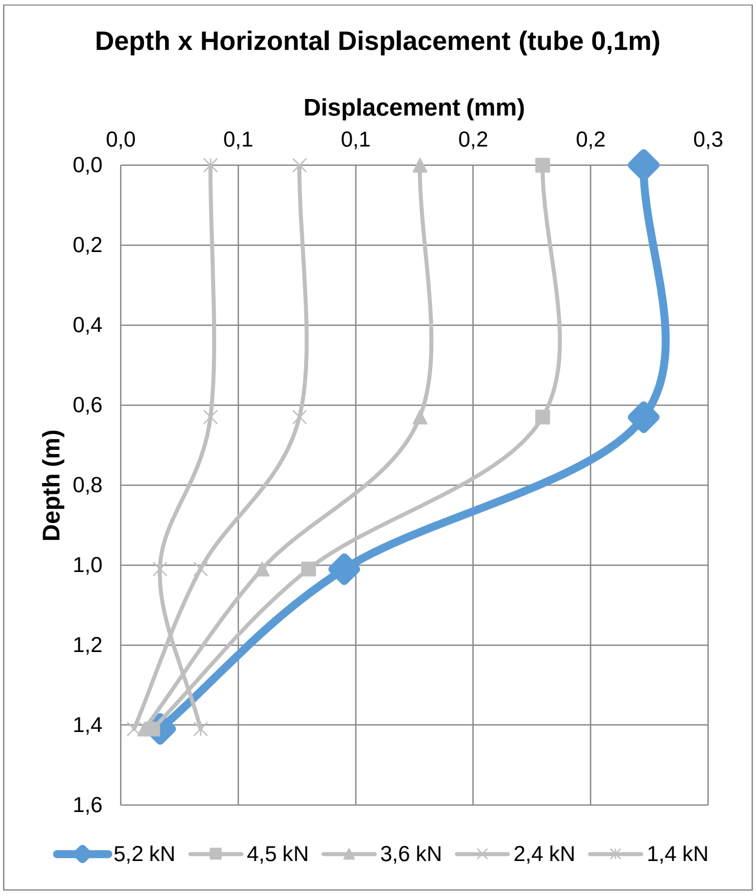

Figure 10. Horizontal displacements in the tube 1 at a radial distance of 0.1 m from the pile; Electro-levels readings

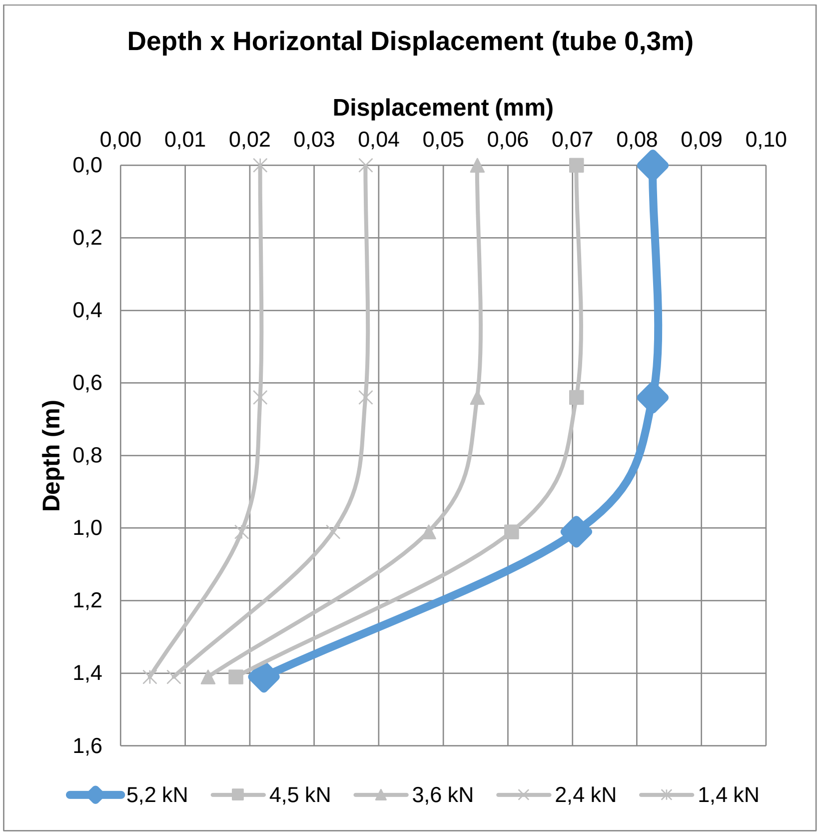

Figure 11. Horizontal displacements in the tube 2 at a radial distance of 0.3 m from the pile; Electro-levels readings

A simplified way to determine the specific deformations in the pile is to use the experimentally measured rotation for two adjacent points and divide by the distance between these two points. Figure 12, and Equations 22 to 25 show the process used to obtain the horizontal deformation in the pile.

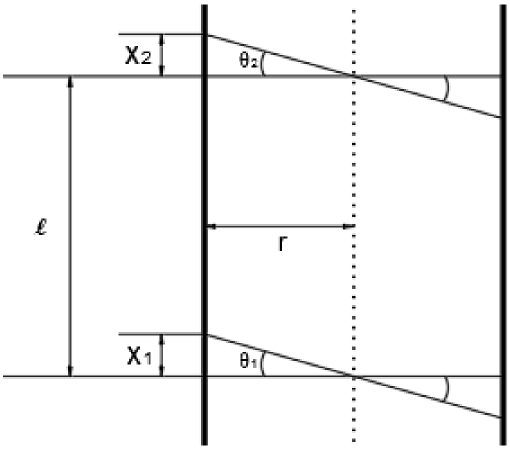

Figure 12. Horizontal deformation of the pile

$$x_1 = \tan(\theta_1) \cdot r \tag{22}$$

$$x_2 = \tan(\theta_2) \cdot r \tag{23}$$

$$\varepsilon = \frac{x_1 - x_2}{l} \tag{24}$$

$$\varepsilon = \frac{r}{l}(\tan\theta_1 - \tan\theta_2) \tag{25}$$

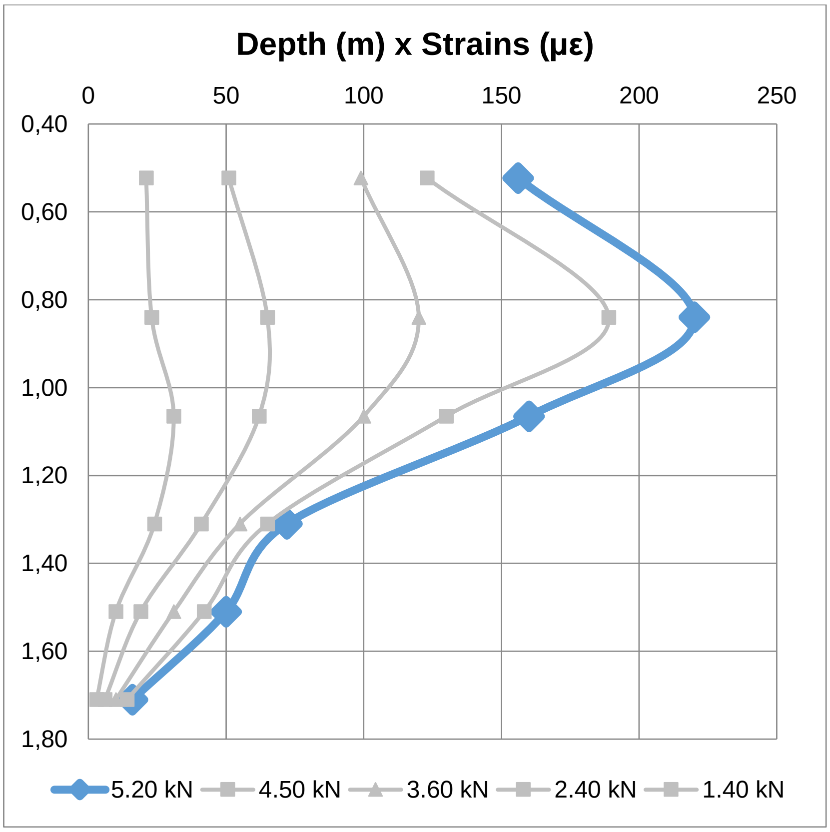

Figure 13. Bending deformations calculated using the results from the Electro-levels

Figure 13 shows the bending deformations of the pile, the deformation value for the depth of 0.685 m was not taken into account to plot the graph, since some inconsistency was detected in the readings from electro-level number 9, as shown previously in Figure 7.

5.4 Pile Stiffness

For piles of reinforced concrete, it is very important to know how the stiffness changes during bending of the pile. Variations in the Young's modulus and moment of inertia of the pile, $E_p I_p$, must be determined for the two pile components, steel and concrete. We adopted $E_p I_p$ values of 1550 kN/m^2 for deformation levels of less than 0,1mm as given by Ramos (1988), with that deformation the concrete began to crack. With deformations of 0,3mm, $E_p I_p$ values of 1000 kN/m^2 were used.

5.5 Representation of rotations

Having established the polynomial function representative of the rotations, the beam theory was used to determine their respective derivatives in order to calculate the bending moments and shear stresses in the pile and the soil reaction. The displacements were determined by integration of the polynomial function.

Displacement: $\int \theta dz + C$
Rotation: $\theta = f(z)$
Bending Moment: $E_p I_p \left[d\theta / dz \right]$
Shear: $E_p I_p \left[d^2\theta / dz^2 \right]$
Soil Reaction: $E_p I_p \left[d^3\theta / dz^3 \right]$

where:

$E_p I_p$ = Stiffness for bending of the pile
θ = Rotation
z = Depth

The integration constant C, which appears in the displacement polynomial, can be determined in two ways: The first is to assume that, at the zero-pressure point, the displacement is also zero. This point can be determined using the polynomial of the reactions. The other way to use the calculated rotations directly from the electro-levels to check the point that corresponds to zero displacement.

5.6 Variations of rotations, bending moments, shear forces and soil reaction modulus with depth

The variations of rotations and the other parameters of interest such as bending moments, shear forces and soil reaction are presented in Figures 14 to 19, results obtained for a horizontal load of 5.2 kN. The representative function obtained for rotations is a Hexic polynomial equation (6th grade); from this function, the stresses in the pile and soil were determined. According to Ramos (1988), a sixth order polynomial fits better than polynomials of the eighth order.

According to the results obtained, a better representation of the stresses was observed, with the introduction of this restrictive condition. The soil reaction modulus increases with depth.

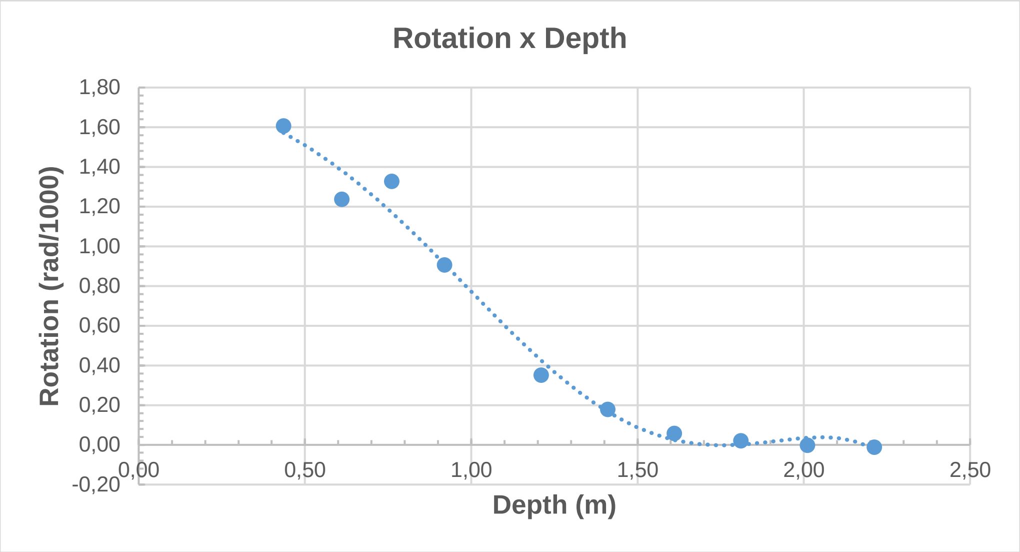

Figure 14. Rotations of the pile

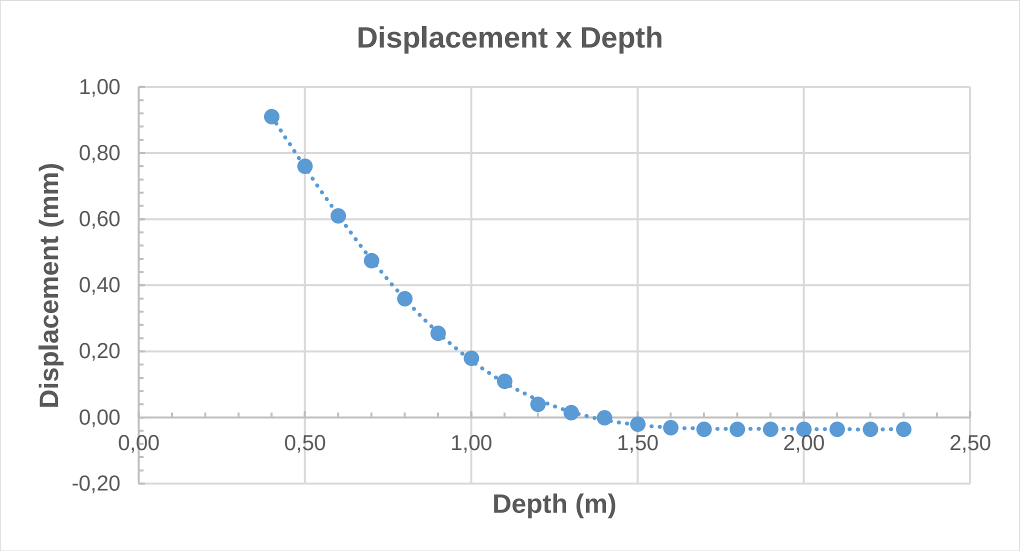

Figure 15. Horizontal displacements of the pile

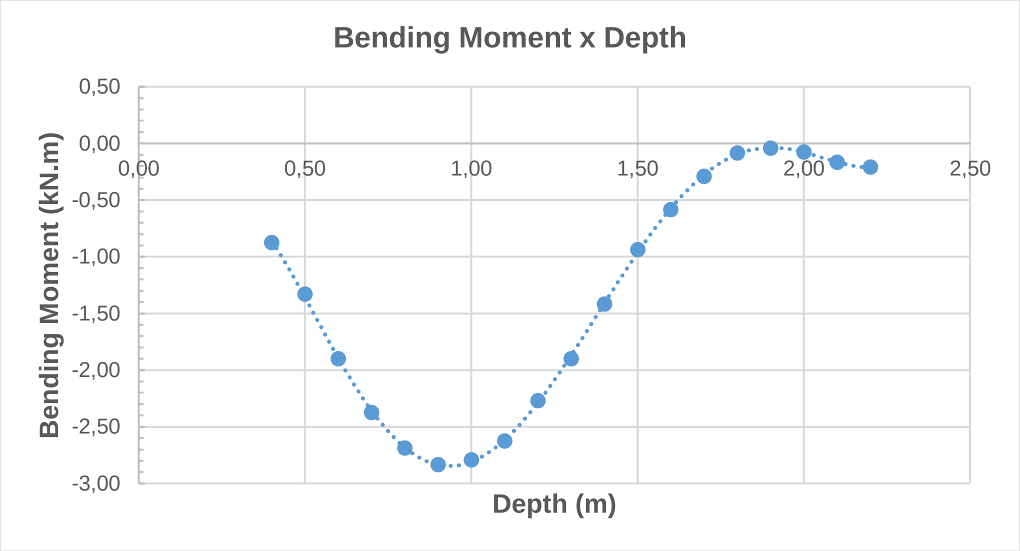

Figure 16. Bending moment of the pile

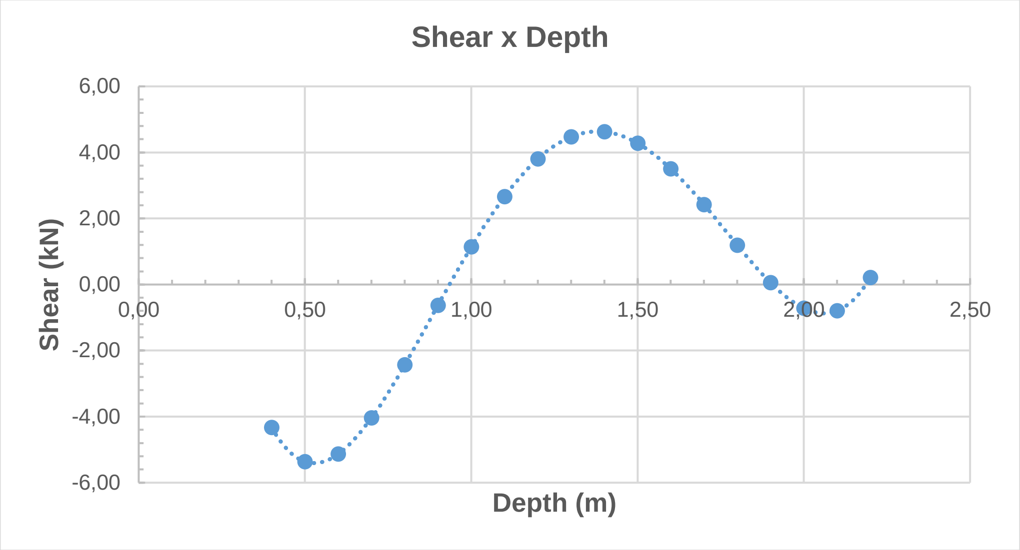

Figure 17. Shear in the pile

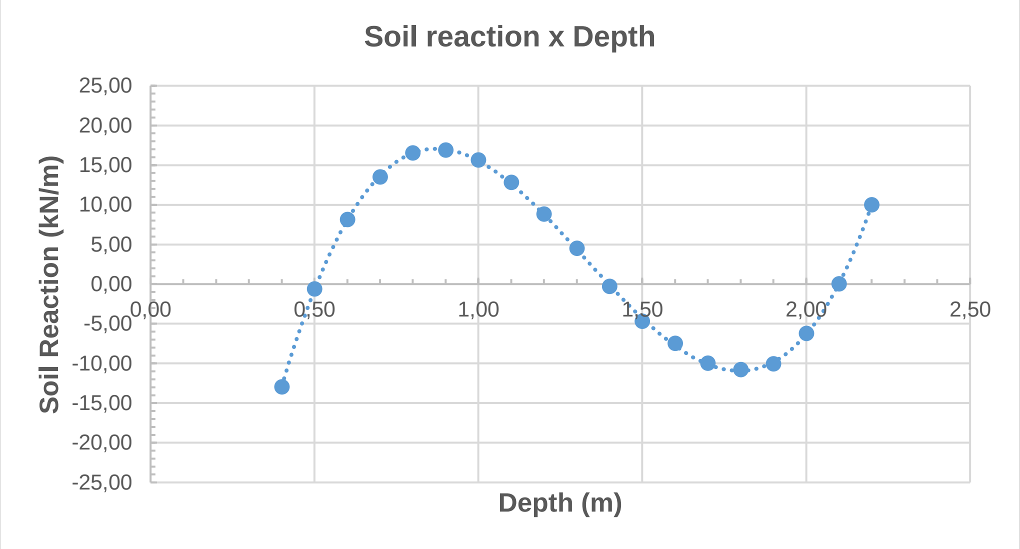

Figure 16. Soil reaction

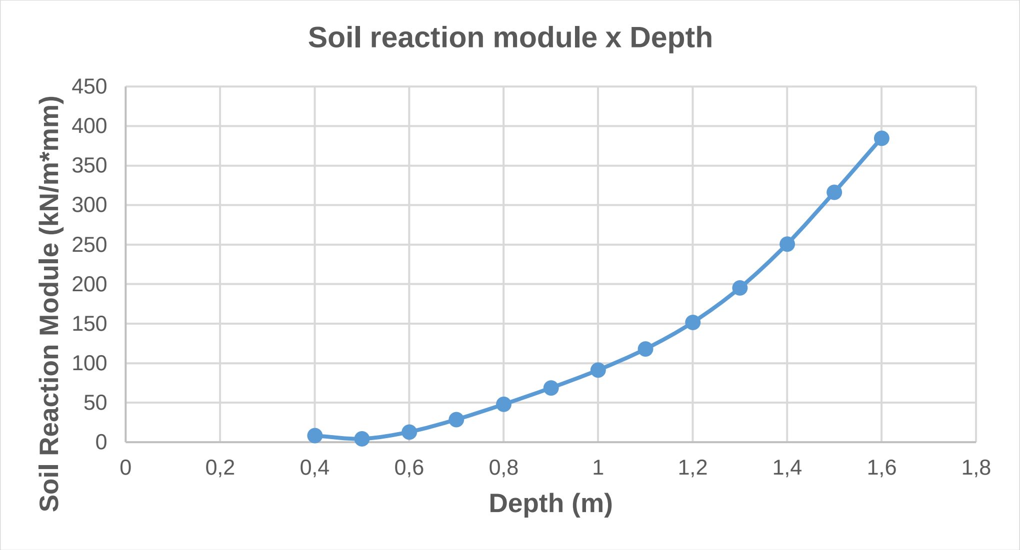

Figure 17. Soil reaction modulus

The elasticity modulus of the soil was calculated for a horizontal load of 5.2 kN using the displacements obtained from the Electro-levels in the tubes installed adjacent the pile, the results were obtained using Mindlin´s integral equation (Vaziri et al, 1982). The results are shown in Table 1 and Figure 18.

Table 1. Young´s Modulus

HORIZONTAL LOAD AT THE TOP OF THE PILE: 5.2KN			
Radial Distance (m)	Depth (m)	Es (kN/m2)	Displacement (mm)
0.100	0.630	34818	0.22253
0.100	1.010	91629	0.09509
0.100	1.410	80173	0.01673
0.300	0.640	60156	0.08242
0.300	1.010	71511	0.07057
0.300	1.410	65495	0.02218

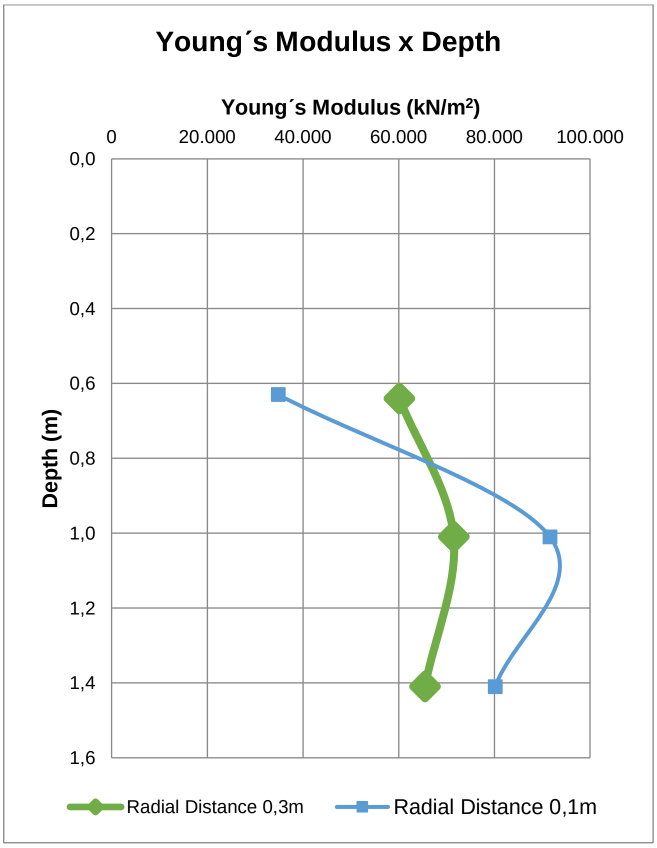

Figure 18. Young´s Modulus

6 CONCLUSIONS

Based on the soil reaction theory, the parameters of interest (bending moments, shear forces and soil reaction) were obtained by successive derivations of the adjusted polynomial data of the rotations. It was found that the restricted sixth degree polynomial adequately characterized the representative function.

The soil reaction modulus obtained experimentally indicated a non-linear variation with depth, and it was verified that the soil presents deformation characteristics proportional to the depth.

Instrumentation with Electro-levels has the advantage of being able to be installed after the pile is already in the ground, are easy to install, cause little disturbance to the pile installation and can be removed for use in other tests, which reduces the real cost of instrumentation.

Electro-levels are feasible for use in commercial pile tests and could also be employed in other soil-structure interaction problems.

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This work would not have been possible without the instrumentation data provided by the Building Research Establishment to professor Rocha Filho.

REFERENCES

Brooms, B.B. (1964). Lateral resistance of piles in cohesive soil, JSMFD, ASCE, v.90, n. SM2, p. 27-65.

Hetenyi, M. (1946). Beams on Elastic Foundation, Theory with Applications in the Fields of Civil and Mechanical Engineeiring, *The University of Michigan Press*, Ann Arbor. 245p.

Poulos, H.G. and Davis, E.H. (1980). Pile Foundation Analysis and Design, *John Wiley and Sons,* New York, NY, USA. 397p.

Price, G. and Wardle, I.F. (1987a). The use of Electro-levels on laterally loaded precast driven piles at Stretton. *Building Research Establisment*, Ground Engineering Division. Contractor Report No. 48.

Price, G. and Wardle, I.F. (1987b). Comparison of vertical and shallow angled raked piles under lateral load. *Building Research Establisment*, Ground Engineering Division. Contractor Report No. 45.

Price, G. and Wardle, I.F. (1987d). Lateral load tests on large bored piles. *Building Research Establisment,* Ground Engineering Division. Contractor Report No. 46.

Price, G. and Wardle, I.F. (1987c). Vertical and Lateral load tests on driven cast in place piles at Yarmouth. *Building Research Establisment*, Ground Engineering Division. Contractor Report No. 47.

Pyke,R.; Beikae,M. (1985). A new solution for the resistance of single piles to lateral loading, Laterally loaded deep foundations: analysis and performance, STP 835, ASTM, Philadelphia, p.3-20.

Ramos, V. (1988). Analysis of vertical piles loaded horizontally on top instrumented with Electro-levels. Dissertation. Puc-Rio. p. 143.

Saran, S. (2009). Analysis and Design of Substructures, Limit State Design. Second Edition, *Oxford & IBH Publising*, New Delhi.

Terzaghi, K. (1955). Evaluation of coefficients of subgrade reaction. *Géotechnique*, vol. 4. P. 297-326

Winkler, E. (1867). Die Lehre von der Elastizitat und Festigkeit, Dominicus, Prague.

Vaziri, H., Simpson, B., Pattin, J.W. and Simpson, L. (1982). Integrated forms of Mindlin´s Equations. *Géotechnique,* No. 3, vol. 32, p. 275-278.